Firms' timing of production with heterogeneous consumers

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Abstract. We revisit endogenous timing in a quantity-setting duopoly game. In the basic model, we show that given strong heterogeneity in consumers' willingness to pay (WTP), and a moderately small consumer segment with low WTP, sequential moving outcomes can appear in equilibrium with the follower enjoying second-mover advantage. Owing to consumer heterogeneity in WTP, there is a local property that a firm's aggressive behavior may lead to a competitor responding more aggressively. Hence, the sequential moves can restrict firms' total outputs to avoid a price collapse, and result in firms' strategic choices that Pareto dominate those under the simultaneous move. We further generalize our results and show that although firms compete in quantity, under some conditions of the demand function, features of strategic complements can appear.

JEL Classification Numbers: D21, L13, M21

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1 Introduction

With the rise of the BRIC economies (Brazil, Russia, India, and China), firms have become more conscious of the potential for profitability in developing countries. One important feature of developing economies is their large income inequality.[1] Consumers with different income levels generally show heterogeneity in their willingness to pay (WTP) when deciding purchases.[2] In developing countries, where consumer heterogeneity in WTP is relatively pronounced, firms always face the following tradeoff: attract the consumers with low WTP by cutting prices, but earn lower revenues from those with high WTP.

This study discusses firms' timing of production when exploiting a new market comprising heterogeneous consumers in their WTP. To this end, we consider the endogenous timing procedure for a two-player game: In the pre-play period, each firm chooses between moving early or moving late and announces its timing decision. In the remaining game (or basic game), if both firms choose to move in the same period, whether early or late, the remaining moves proceed simultaneously. If each firm chooses to move in a different period, the remaining moves proceed sequentially.[3]

To capture consumer heterogeneity in WTP and explicitly derive our main results, we first consider a simple market structure with a consumer segment whose WTP is far lower than that of the remaining consumers. We call this segment L consumers. When the price is low, L consumers swarm into the market and immediately start purchasing, giving rise to a "long-tail" part on the inverse demand curve. Hence, this suggests two potential market statuses: (1) firms give up L consumers and maintain a premium price (market separation) or (2) make their products affordable to L (market integration). In this setting, two types of equilibrium outcomes can exist under different market statuses: under market separation, both firms choose small outputs; under market integration, both firms choose large outputs. Endogenizing firms' timing enables the timing equilibrium outcomes to interact with different market statuses, giving rise to significantly different results compared with the standard endogenous timing game

[1] Benjamin, Brandt and Giles (2005) show that although China has seen impressive growth in recent years, the absolute living standards of the poor have declined; the rising tide has not lifted all boats. Sicular, Yue, Gustafsson and Li (2007) discuss the urban-rural income gap and its contribution to inequality in China. Chen and Ravallion (2007) study other developing countries and find a marked urbanization of poverty. Wang and Woo (2011) find that 63% of the unreported income went to 10% of the richest urban households, and the actual Gini coefficient is much higher than the official one.

[2] One good example is the low-income consumers' demand for counterfeit products which are low-priced but always similar functionally to authentic ones. Qian (2008) empirically shows that higher income inequality may give rise to more demand for counterfeit products. Needless to say, theoretical studies also focus on how income inequality results in consumer heterogeneity in WTP (e.g., Tirole 1988, pp. 143–144).

[3] Hamilton and Slutsky (1990) insightfully refer to such a timing game as "the extended game with observable delay." In this timing game, we potentially have two simultaneous subgames and two sequential subgames.

under quantity competition.

We show that even with quantity competition, given strong heterogeneity in consumers' WTP and a moderately small consumer segment with low WTP, firms' strategic choices under the sequential moves (either firm leading and the other one following) Pareto dominate those under the simultaneous moves, giving rise to sequential timing outcomes in equilibrium. The intuitive explanation for this is as follows: Owing to the existence of L consumers, firms may exercise their option to increase production such that the market price is driven low enough so that L consumers can afford their products. Therefore, even though there is quantity competition, each firm's reaction function jumps upward when the rival's output exceeds a certain threshold, implying that firm strategies become complements locally at the jump point. When firms move sequentially, the follower's locally aggressive response (upward jump) may force the leader to restrict output so that the total supply is maintained at a low level to prevent the price from collapsing. However, when firms move simultaneously, given that their respective outputs are not mutually restricted, overproduction and price collapse are more likely to occur (if one firm chooses a large output, its rival simultaneously responds with a large output). When supplying to L consumers is not profitable, sequential moving allows both the leader and follower to restrict total outputs, maintain premium pricing, and stop supplying L consumers.

The sequential timing outcomes stem from the upward shift in the reaction functions, which causes local distortions in firms' strategic interactions. This differs from a general quantity competition in which a player's aggressive behavior always results in a mild response from its rival. We use a segmented market structure in our basic setting that provides linearity, enabling us to explicitly derive our primary results and substantially simplify the analysis. To add robustness to our analysis and remove the effect of a segmented and linear structure, we also use a general demand function to provide conditions under which a discontinuous reaction function with an upward shifting point is presented. We further demonstrate that our results in the basic model can also hold even if consumers are not so extreme as to allow for segmentation.

The results about firms' sequential moving outcomes have two implications for the timing of new product release. First, such timing is closely related to the market's structural elements such as market size or population of consumers. Second, a firm may abandon its pioneering position to mitigate head-to-head competition and realize a higher profit. Some empirical works seem to be consistent with the above arguments. First, Mahajan and Muller (1996) study the IBM case and find that decisions to introduce a new generation product as soon as possible or to delay it until maturity are closely affected by the relative size of the potential market. Second, Krider and Weinberg (1995) show that in the film industry, some movie companies would rather delay a debut and let their rivals take action first to avoid head-to-head competition. Although the structure of the personal computer and film industries are not exactly the same as those described in this paper, there is some common logic.[4]

[4] The timing of new product release is a good example that fits our setting because in the initial

Coming to the theoretical literature, we find that the study by Hamilton and Slutsky (1990) is among the earliest attempts to discuss firms' endogenous timing problem.[5] Their core results imply that under quantity competition with each firm's continuous reaction function, equilibrium should have a simultaneous moving outcome. In our study, although firms compete in quantity, because of the strong heterogeneity in consumers' WTP, the reaction function presents an upward shifting feature, which endows the quantity competition with features of both strategic substitutes and strategic complements. Our results are partially consistent with Hamilton and Slutsky (1990) in the sense that both their timing outcomes–the simultaneous timing outcome under strategic substitutes and the sequential moving outcome under strategic complements–may occur in quantity competition.

The sequential timing outcomes follow from the fact that strategies in quantity competition may become complements under some conditions. Such distortion in firms' strategic interactions is also demonstrated by other studies. For example, Bulow, Geanakoplos and Klemperer (1985) show that a dominant firm with high market share regards products as strategic complements when the demand curve is with a constant elasticity. Amir and Grilo (1999) study quantity competition using the approach of super-modular games and identify conditions directly on the demand and cost functions under which strategies become complements. Amir, Amir and Jin (2000) consider firms' strategic cost-reducing investment and spillover. Strategies of investment become complements for a firm when its rival's spillover generates enough net benefits. In Monaco and Sabarwal (2012), a simple example of Cournot duopoly with unilateral spillover is used to show that strategic distortion, contingent on the slope of spillover function, may occur. Some of the above studies also discuss the endogenous timing problem and demonstrate the existence of sequential timing outcomes (Amir and Grilo 1999; Amir, Amir and Jin 2000).[6] Our study complements the literature in that the

stage, firms usually decide quantity rather than price. Unfortunately, there are limited empirical works investigating the direct relations among firm's timing in releasing new products, consumers' heterogeneity in WTP and market profitability. For one, as stated in Ishibashi and Matsushima (2009), it is difficult for researchers to access firms' profit data; thus, it is hard to measure the magnitude of the profitability of a market. For another, Urban, Carter, Gaskin and Mucha (1986) find that in some industries, firms can delay their actions to different extents, i.e., being the first follower or the second follower means a different market share. Despite these difficulties, some indirect empirical works are consistent with the current paper in some related results. Axarloglou (2003) analyzes the cyclical nature of the timing of new product introductions in U.S. manufacturing. Mukherjee and Kadiyali (2008) discuss the release timing in the DVD market and Engelstatter and Ward (2013) study the timing in the video games market.

[5] For other parallel studies that discuss endogenous timing problems and the strategic rivalry between firms, see Yang, Luo and Wu (2009) and Matsumura and Ogawa (2009).

[6] The sequential timing outcomes may also exist under asymmetric information. Normann (2002) considers a case wherein one firm is privately informed about the state of demand, while the other firm remains uninformed. Mailath (1993) is an early work of Normann (2002) that studies almost the same market model but applies the extended game with action commitment. Several others consider a situation where market uncertainty vanishes if firms choose to delay action (e.g., Spencer and Brander 1992; Sadanand and Sadanand 1996).

strategic distortion comes from consumers' heterogeneity in WTP.

The essential driving force of our main results is each firm's discontinuous reaction function. Generally, as shown by some studies, firms may develop discontinuous strategic responses when consumers can be segmented into groups when the market presents different statuses. Adner and Zemsky (2005) consider a Cournot oligopoly wherein firms with new technologies compete with those with established ones for two discrete consumer segments. The market presents two statuses wherein new technologies serve only isolated segments or break the boundaries of different segments. Ishibashi and Matsushima (2009) consider two types of firms–those producing high-end products and those producing low-end ones and heterogeneous consumer segments–consumers buying only high-end products and those who are indifferent between high- and low-end products.[7] The market presents two statuses wherein high-end firms supply both segments or only the one that prefers high-end products. In this study, the discontinuous reaction functions do not necessarily come from segmentable consumer groups (as shown in Section 5). The process of endogenizing timing based on discontinuous reaction functions add several new insights to the literature.

The remaining paper is organized as follows: Section 2 introduces a basic model using the linear demand function with consumers' heterogeneity in WTP. Section 3 details the equilibrium analysis in each subgame. Section 4 derives the equilibrium timing outcomes in the pre-play period and elaborates on the related propositions. Section 5 generalizes the results in our basic model by considering the general demand case. Section 6 concludes the paper.

2 A basic model

Let us consider a homogeneous products duopoly. Let q_i and q_j denote the output levels of firm i and firm j, where i, $j = 1, 2$. To show how consumers' heterogeneity in willingness to pay (WTP) affects firms' timing outcome and get rid of asymmetry from the firms' side, we assume that firms are symmetric in that they don't face asymmetry in information or cost structures. For simplicity, we assume the marginal production cost is zero and that there is no fixed cost.

In the basic model, we consider a simple case wherein the market comprises two groups of consumers–one group of H consumers with high WTP (or high-end market) and the other group of L consumers with low WTP (or low-end market). We assume that H consumers' WTP is uniformly distributed on $[0, 1]$ and that of L consumers is uniformly distributed on $[0, a]$, where $0 < a < 1$. The size of the high-end market and that of the low-end market are 1 and $b > 0$, respectively. The segmentation of

[7] Roy (2014) examines durable goods in a market with consumers' heterogeneity in WTP. The author finds that consumers' heterogeneity may result in inefficient allocation where high-valuation buyers buy low-quality goods.

different consumer types and its uniform distribution enable us to construct a linear demand structure, which helps us to better track the targeting mechanism from a clear economic perspective. The inverse demand function is given as follows:[8]

$$P(q_1+q_2)=\begin{cases}1-q_1-q_2 & if\ q_1+q_2\leq 1-a,\\ \dfrac{a(1+b-q_1-q_2)}{a+b} & if\ q_1+q_2>1-a,.\end{cases} \tag{1}$$

If $1-q_1-q_2 \geq a$, both firms supply to only H consumers, implying a *market separation.* If $0<1-q_1-q_2<a$, because L consumers can also afford the products, P is decided by the total demand from both H and L consumers, implying *market integration.* Notice that this demand structure is not the essential driving force of our main results. To add robustness to our analysis and remove the effect of a kinked and linear structure, besides the current case wherein consumers are strictly segmented, we also consider a general demand case in which consumers' heterogeneity in WTP is captured but not as extremely as in the current model, and find that our results still hold. The general demand case is discussed in Section 5.

Each firm's profit maximization problem is given by

$$\max_{q_i}\pi_i(q_i,q_j)\equiv P(q_i+q_j)q_i, i,j=1,2,i\neq j, \tag{2}$$

from which the reaction function is derived as follows:

$$R_i(q_j)=\begin{cases}R^S(q_j)\equiv\dfrac{1-q_j}{2} & if\ q_j\leq q_J\equiv 1-a-\sqrt{a(a+b)},\\ R^I(q_j)\equiv\dfrac{1+b-q_j}{2} & if\ q_j\geq q_J.\end{cases} \tag{3}$$

where subscript S and I denote *separation* and *integration* respectively (see appendix for calculation). q_J is the threshold at which firm i is indifferent between $R^S(q_j)$ and $R^I(q_j)$. Since $R^S(q_J)<R^I(q_J)$, the reaction function presents an upward shifting feature. With the existence of L consumers, q_j that just surpasses the threshold triggers firm i's aggressive behavior of oversupply, which drives the price low enough to cause market integration. Therefore, although firms compete in quantity so that the strategic variables are local substitutes (for $q_j\leq q_J$ or $q_j>q_J$), they become local complements at the threshold value, wherein L consumers start purchasing.

To frame the endogenous timing issue, we follow the timing structure proposed by Hamilton and Slutsky (1990) and construct an "extended game with observable delay": In the pre-play period, each firm faces strategic options between moving early or late. The timing decisions are then observed by both firms. The remaining game (or

[8] The derivation of the inverse demand function is straightforward, going by Ishibashi and Matsushima (2009). We show the procedure in appendix.

basic game) proceeds according to the timing decided in the pre-play period. If both firms choose to move in the same period, whether early or late, the remaining moves proceed simultaneously. If each firm chooses to move in a different period, the remaining moves proceed sequentially. Therefore, we potentially have two simultaneous subgames and two sequential subgames. The price is decided after both firms have chosen their quantities. The two types of the basic game (simultaneous or sequential) share the same reaction function as denoted by Eq. (3).

3 Equilibrium analysis

In the basic game, the timing decisions made in the pre-play period give rise to two cases as follows: I, firms move simultaneously, whether early or late; II, firms move sequentially, with either firm being the leader and the other, the follower. For each case, in equilibrium, the firms can either choose outputs of a low level, which results in *market separation*, or those of a high level, which results in *market integration*. The equilibrium is solved by backward induction. Throughout this analysis, from Section 2 to Section 4, equilibria are derived by first-order conditions, and the solutions satisfy the unique pure-strategy subgame perfect equilibria. Before we derive the equilibrium outcomes for each subgame, we introduce the following assumptions of our basic model:

Assumption 1 *(i) Consumer heterogeneity in WTP is strong enough. Formally, $a \leq 1/4$.*

(ii) In each simultaneous subgame, there could exist multiple equilibria. We apply payoff dominance to select the unique equilibrium outcome.

The first assumption is made for tractability of both market statuses. If L consumers' WTP is too high ($a > 1/4$), both firms always have an incentive to supply the low-end market, whether they move simultaneously or sequentially; thus, we cannot have the market separation status. The second assumption is technically required because we need to select the unique equilibrium in each subgame. An alternative way to understand this assumption is that firms can coordinate between themselves on their preferred equilibrium. Notice that this assumption provides robustness to our study because the main propositions becomes even more likely to hold if the payoff dominated pair is selected. We will refer to this point when introducing Proposition 1.

Case I : Firms move simultaneously

We use subscript c to denote the Cournot case wherein firms move simultaneously, whether early or late. Based on the reaction function in Eq. (3), in the symmetric game, two types of equilibrium candidates can be derived by solving $R^S(q_j) = q_j$ and $R^I(q_j) = q_j$:

$$\hat{q}_c^S \equiv \frac{1}{3},\ \hat{q}_c^I \equiv \frac{1+b}{3}, \tag{4}$$

and the corresponding equilibrium profits

$$\pi_c^S \equiv \frac{1}{9},\ \pi_c^I \equiv \frac{a(1+b)^2}{9(a+b)}. \tag{5}$$

To guarantee stability, each type of the above equilibrium candidates must satisfy the corresponding conditional inequality in Eq. (3). Notice that both equilibrium candidates could coexist within the same parameter range. Specifically, when $(4+3a-3\sqrt{8a+a^2})/2 \leq b \leq (4-12a)/(9a)$, both firms may choose a low output level $\hat{q}_c^S$ to make the price unaffordable for L consumers; they may also choose a high output level $\hat{q}_c^I$ to make the price low enough to target both H and L consumers. Technically, we need Assumption 1 (*ii*) to select the unique equilibrium in this subgame. $\pi_c^S = \pi_c^I$ specifies a threshold $b = (1-2a)/a$ which is larger than the upper bound value of the range wherein both equilibrium candidates coexist. Hence, when $(4+3a-3\sqrt{8a+a^2})/2 \leq b \leq (4-12a)/(9a)$, $(\hat{q}_c^S, \hat{q}_c^S)$ dominates $(\hat{q}_c^I, \hat{q}_c^I)$ and is thus selected as the unique equilibrium outcome. The conditions under which the corresponding equilibrium outcomes exist are summarized in the following lemma (see appendix for the calculation):

Lemma 1 *In each simultaneous subgame, given strong heterogeneity in consumers' WTP, firms' equilibrium quantities* (q_i, q_j) *are*
(i) $(\hat{q}_c^S, \hat{q}_c^S)$ *if* $b \leq (4-12a)/(9a)$*;*
(ii) $(\hat{q}_c^I, \hat{q}_c^I)$ *if* $b > (4-12a)/(9a)$*.*

From Lemma 1, when firms move simultaneously and if the size of the low-end market is large enough, both firms choose equilibrium outputs of a high level to drive down price and supply to both H and L consumers. In other words, given that one firm acts aggressively, it is always better for the other one to react aggressively. However, it is not always profitable for firms to supply the L consumers. For $(4-12a)/(9a) < b \leq (1-2a)/a$, although focusing only on H consumers is actually a better choice for both firms ($\pi_c^S \geq \pi_c^I$), both firms still oversupply and enter the low-end market. Notice that such "unprofitable entry" cannot be avoided even though we assume the firms can coordinate well as stated in Assumption 1 (*ii*).[9]

Case II: Firms move sequentially

We call the firm moving early the leader and the one moving late the follower. Owing to symmetry, we naturally have two sequential subgames wherein either firm becomes the leader. In the basic game, the follower's reaction is as denoted in Eq. (3). Because of the follower's locally upward shifting reaction function, when the leader's output reaches q_J, the follower raises its own output from $(1-q_J)/2$ to $(1+b-q_J)/2$, which results in a substantial decrease in price. By doing so, the follower enables L consumers

[9] For $(4-12a)/(9a) < b \leq (1-2a)/a$, given that one firm chooses $\hat{q}_c^S$, the other one always unilaterally deviates by choosing an output larger than $\hat{q}_c^S$. That is, $q' = \arg\max_q P(\hat{q}_c^S + q)q > \hat{q}_c^S$.

to afford the products and thereby enters the low-end market. Although loses out because of a lower price, the follower maintains the same profit level by increasing the quantity sold.

The leader solves the following profit maximization problem, anticipating the follower's reaction:

$$\max_{q_l} \pi_l[q_l, R_i(q_l)] = \begin{cases} \pi_l^S(q_l) \equiv \dfrac{1-q_l}{2} q_l & if\ q_l \leq q_J, \\ \pi_l^I(q_l) \equiv \dfrac{a(1+b-q_l)}{2(a+b)} q_l & if\ q_l \geq q_J. \end{cases} \tag{6}$$

This implies that $\pi_l^S(q_J) > \pi_l^I(q_J)$ for any $0 < a < 1$, $b > 0$. The leader's profit function has a downward jump at q_J, which is caused by the follower who substantially changes its response by raising output. At this threshold, the sudden decrease in price harms the leader who cannot raise quantity immediately.

The leader potentially has two types of interior equilibrium candidates which are derived from $\max_{q_l} \pi_l^S(q_l)$ and $\max_{q_l} \pi_l^I(q_l)$. The leader's and follower's equilibrium outputs are

$$\hat{q}_l^S \equiv \frac{1}{2},\ \hat{q}_f^S \equiv \frac{1}{4},\ \hat{q}_l^I \equiv \frac{1+b}{2},\ \hat{q}_l^I \equiv \frac{1+b}{4}, \tag{7}$$

and the corresponding equilibrium profits are

$$\pi_l^S \equiv \frac{1}{8},\ \pi_f^S \equiv \frac{1}{16},\ \pi_l^I \equiv \frac{a(1+b)^2}{8(a+b)},\ \pi_f^I \equiv \frac{a(1+b)^2}{16(a+b)}. \tag{8}$$

Each type of the above equilibrium candidates must satisfy the corresponding conditional inequality in Eq. (3). Different from the simultaneous subgames, because of the leader's discontinuous profit function, we possibly have q_J as another corner equilibrium candidate. The leader's and follower's outputs are

$$\hat{q}_l^C \equiv q_J,\ \hat{q}_f^C \equiv \frac{1-q_J}{2}, \tag{9}$$

with the corresponding equilibrium profit

$$\pi_l^C \equiv \frac{(1-q_J)q_J}{2},\ \pi_f^C \equiv \frac{(1-q_J)^2}{4}, \tag{10}$$

where superscript C denotes a corner solution. FIGURE 1 depicts the situation wherein q_J becomes the leader's global profit maximizer. Specifically, the leader cannot reach $\hat{q}_l^S$ before the price collapse occurs (i.e., $q_J \leq \hat{q}_l^S$), and its profit at the jump point q_J is larger than that at $\hat{q}_l^I$ (i.e., $\pi_l^C > \pi_l^I$). Given the mathematical complexity involved, we cannot explicitly derive the threshold value of b that satisfies $\pi_l^C = \pi_l^I$. Therefore, we denote this threshold value by $b(a)$ numerically. The following lemma summarizes the conditions under which the corresponding equilibrium outcomes exist in each sequential

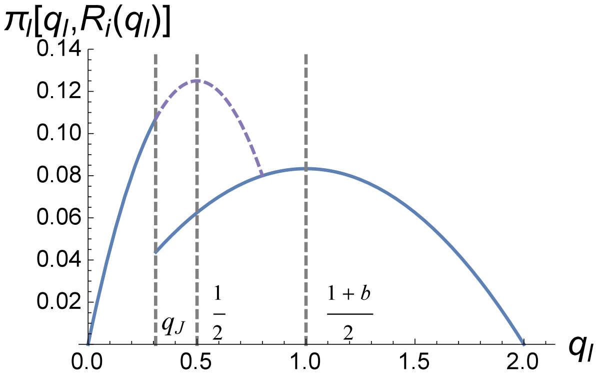


FIGURE 1 The leader's profit function in each sequential subgame when $a = 0.2$ and $b = 1$

TABLE 1
Threshold values of $b(a)$

a	0.02	0.04	0.06	0.08	0.10	0.12
$b(a)$	30.238	14.235	8.899	6.229	4.625	3.554
$(4-12a)/(9a)$	20.889	9.778	6.074	4.222	3.111	2.370
$(1-4a)/(4a)$	11.500	5.250	3.167	2.125	1.500	1.083

a	0.14	0.16	0.18	0.20	0.22	0.24
$b(a)$	2.787	2.210	1.759	1.396	1.095	0.839
$(4-12a)/(9a)$	1.841	1.444	1.136	0.889	0.687	0.519
$(1-4a)/(4a)$	0.786	0.563	0.389	0.250	0.136	0.042

game (see appendix for calculation):

Lemma 2 *In each sequential subgame, given strong heterogeneity in consumers' WTP, firms' equilibrium quantities (q_l, q_f) are*
(i) $(\hat{q}_l^S, \hat{q}_f^S)$ *if* $b \leq (1-4a)/(4a)$;
(ii) $(q_J, (1-q_J)/2)$ *if* $(1-4a)/(4a) < b \leq b(a)$;
(iii) $(\hat{q}_l^I, \hat{q}_f^I)$ *if* $b > b(a)$.

We set a at 0.02 intervals and find that $b(a) > (1-4a)/(4a)$ for any $a < 1/4$.[10] The result is summarized in TABLE 1.

When the size of the low-end market is too small (i.e., $b \leq (1-4a)/(4a)$), firms adopt locally optimal outputs of a low level $(\hat{q}_l^S, \hat{q}_f^S)$ to segment the market and supply to only H consumers. When the size of the low-end market is too large (i.e., $b > b(a)$), on the other hand, targeting both H and L consumers is better than focusing only on H consumers. In this case, firms adopt locally optimal outputs of a high level $(\hat{q}_l^I, \hat{q}_f^I)$ to integrate the market. The outputs of $(q_J, (1-q_J)/2)$ are selected when the low-end market is of an intermediate size. Although firms compete in quantity so that the

[10] This relation holds true even if we set a at 0.001 intervals and check the threshold values of b at each a.

TABLE 2
The payoff matrix in the pre-play period

firm 1 \ firm 2	early	late
early	(π_c, π_c)	(π_l, π_f)
late	(π_f, π_l)	(π_c, π_c)

strategic variables seem to be strategic substitutes, because of the upward shift in the follower's reaction function, the leader's output at the corner point q_J can be smaller than the follower's, which is indicative of strategic complements; that is, given the leader's aggressive behavior, the follower too reacts aggressively. We will confirm this when we derive the equilibrium outcomes in the full game.

Remark 1 *When the leader and the follower choose* $(q_l, q_f) = (q_J, (1 - q_J)/2)$ *in equilibrium and if* $q_J < 1/3$*, the leader's output is smaller than the follower's.*

4 Endogenous timing in the pre-play period

In the pre-play period, firms endogenously decide their timing, anticipating their resultant profits in the following basic game. The payoff matrix in the pre-play period is depicted in TABLE 2. The following proposition describes three parameter ranges, according to which three types of timing outcomes in equilibrium are presented (see appendix for the calculation):

Proposition 1 *Consider a quantity-setting duopoly competition. Given strong heterogeneity in consumers' WTP,*

(i) if $0 < b \leq (4 - 12a)/(9a)$*, firms move simultaneously in the basic game, and the market is separated;*

(ii) if $(4 - 12a)/(9a) < b \leq b(a)$*, firms move sequentially in the basic game with firm i being the leader,* $i = 1$ *or* 2*, and the market is separated;*

(iii) if $b > b(a)$*, firms move simultaneously in the basic game, and the market is integrated.*

Within the ranges of (*i*) and (*iii*), firms always face a consistent market status of either a separation or an integration, whether they move simultaneously or sequentially. Hence, the timing outcomes are quite standard and consistent with most literature on endogenous timing (e.g., Hamilton and Slutsky, 1990; Amir and Grilo, 1999).

We focus on the sequential moving outcome in (*ii*).[11] In this case, if the size of the low-end market is moderately small, the market integrates when firms move simultaneously but separates when firms move sequentially. When firms move simultaneously,

[11] We set a at 0.02 intervals and find that $b(a) > (4 - 12a)/(9a)$ for any $a < 1/4$ (see TABLE 1).

the lower bound of b (i.e., $(4-12a)/(9a)$) triggers their incentives to oversupply and enter the low-end market. On the other hand, the upper bound of b (i.e., $b(a)$) causes firms' "unprofitable entry" as discussed in the simultaneous subgames. When firms move sequentially, the leader has to commit to a small output level, which drives up price and prevents entry into the low-end market. Then, the sequential moves act as a tool of commitment that enables firms to restrict their total outputs and therefore end up with their strategy choices that Pareto dominate those from the simultaneous moves.[12]

Notice that Assumption 1 (ii) diminishes the possibility under which the sequential moving outcome appears. This is because we select the equilibrium that brings firms higher profits when moving simultaneously, which increases their incentives to deviate from the sequential moving outcome. Thus, the range in Proposition 1 (ii) provides a strong condition for the existence of the sequential moving outcome. Without Assumption 1 (ii), firms have less incentive to move simultaneously, making the sequential moving outcome more likely to occur.

FIGURE 2 depicts firms' reaction functions and isoprofit curves when $a = 0.2$, $b = 1$. The reaction curves intersect at $(\hat{q}_c^I, \hat{q}_c^I)$. The Pareto superior set relative to the Cournot equilibrium output level is denoted by the shaded areas. The lower left and upper right areas denote the separation and integration statuses respectively. Each firm's reaction curve enters the Pareto superior set at $q_i \leq q_J$ and reaches the maximum profit level at q_J.

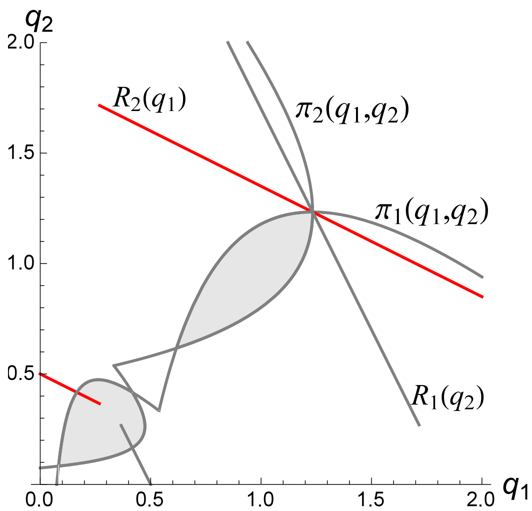


FIGURE 2 Firms' reaction functions and Pareto superior set

As a necessary condition for the existence of the sequential moving outcome, the follower earns a higher profit than the leader in the range of Proposition 1 (ii), which is summarized by the following corollary:

[12] Ishibashi and Matsushima (2009) use a similar setting to demonstrate how the low-end market negatively affects firms' profitability.

Corollary 1 *If* $(4-12a)/(9a) < b \leq b(a)$, *in equilibrium, the follower earns a higher profit than the leader (the second-mover advantage).*

The intuitive reasoning for this follows from the follower's upward shifting reaction function. The leader has to restrict its output to q_J to maintain the price high enough. Conversely, the follower need not worry about the price collapsing because it raises the output in one stroke to compensate for losses from dropping the price.[13] Notice that in Proposition 1 (*ii*), there exist multiple timing equilibria wherein either firm leading and the other following. At each equilibrium, firms coordinate their timing choices to a sequential one, although facing the strategic uncertainty that either of them could turn to its own preferred equilibrium strategy, moving late, which implies a coordination failure.[14]

5 General demand

One sufficient condition to derive the two results in our basic model is consumers' strong heterogeneity in WTP such that demand shows a pronounced increase when price is low. Such a demand structure results in an upward shift of the reaction function and a sudden drop in price when firms move sequentially, from which the resulting profit function is globally nonconcave and has multiple locally optimal values. The kinked demand structure in our basic model is just one example whose linearity enables us to explicitly derive equilibrium conditions, which simplifies our analysis. With respect to the robustness of our findings, we believe that our results can also hold under alternative demand structures other than the specific kinked linear one.

To this end, we consider a homogeneous products duopoly with inverse demand function $P(\cdot)$. Firms i and j's quantities are denoted by q_i and q_j respectively, for $i, j = 1, 2$. We only consider the residual demand faced by firm i given firm j's quantity, which means q_j is treated as an exogenous parameter. We first make the following standard assumptions:

Standard Assumptions

- $P(q_i + q_j)$ is twice continuously differentiable with $P'(q_i + q_j) < 0$.
- $P(q_i + q_j)$ intersects the horizontal axis when $q_i + q_j$ is large, or $\exists \bar{Q}$ such that $P(q_i + q_j) = 0$ for $q_i + q_j \geq \bar{Q}$.

Following the basic setting and for simplicity, we generalize the marginal cost to zero. Firm i's profit $\pi_i(q_i, q_j) = P(q_i + q_j)q_i$. To focus on how the shape of $P(\cdot)$ affects

[13] For literature about the second mover advantage, see Amir and Stepanova (2006) and Julien (2011).

[14] Because the game is symmetric, we cannot select the unique equilibrium by the criterion of risk dominance in Harsanyi and Selten (1988). The equilibrium selection problem can be solved if we consider asymmetry in firms' cost or information structure.

the firm i's reaction function, it is helpful to use iso-profit functions given by

$$I(q_i, \bar{\pi}) = \frac{\bar{\pi}}{q_i}, \tag{11}$$

where $\bar{\pi} > 0$. The iso-profit functions are depicted as parallel curves in FIGURE 3 where $\bar{\pi}$ increases to the upper right. We then define the following:

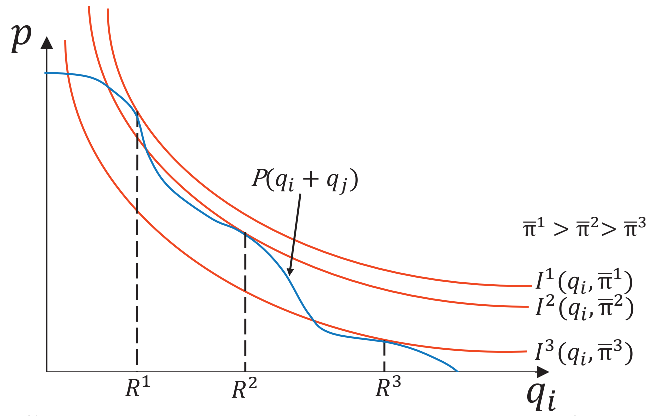


FIGURE 3 Inverse demand curve and iso-profit curves

Definition 1 *Given q_j, $P(q_i + q_j)$ and $I(q_i, \bar{\pi})$ are tangent at $q_i = R$, if $q_i = R$ solves*

$$P(q_i + q_j) = I(q_i, \bar{\pi}) \text{ and } \frac{\partial P(q_i + q_j)}{\partial q_i} = \frac{\partial I(q_i, \bar{\pi})}{\partial q_i}. \tag{12}$$

From Definition 1 and the expression for $I(q_i, \bar{\pi})$, the tangent point R must satisfy the following condition:

$$P(q_i + q_j) = I(q_i, \bar{\pi}) = -\frac{\partial I(q_i, \bar{\pi})}{\partial q_i} q_i = -\frac{\partial P(q_i + q_j)}{\partial q_i} q_i. \tag{13}$$

Eq. (13), or the tangent condition, specifies R with a certain profit level $\bar{\pi}$.[15] As depicted in FIGURE 3, there may exist more than one tangent point, contingent on the shape of $P(q_i + q_j)$. We denote each tangent point by R^k. We find that the tangent condition and $P(q_i + q_j)$'s locally strong log-concavity decides a local maximizer, which is summarized as follows:[16]

Lemma 3 *Suppose the standard assumptions hold, if $P(q_i+q_j)$ and $I(q_i, \bar{\pi})$ are tangent at $q_i = R^k(q_j)$, and $\exists \epsilon > 0$ such that $P(q_i + q_j)$ is strongly log-concave $\forall q_i \in (R^k - \epsilon, R^k + \epsilon)$, then $R^k(q_j)$ is a local maximizer.*

[15] The tangent condition is equivalent to the first order condition.

[16] $P(\cdot)$ is log-concave if $\log P(\cdot)$ is a concave function. Formally, $[\partial^2 P(q_i+q_j)/\partial q_i^2]P(q_i+q_j) - [\partial P(q_i+q_j)/\partial q_i]^2 < 0$.

Proof. Because $P(q_i + q_j)$ and $I(q_i, \bar{\pi})$ are tangent at $q_i = R^k(q_j)$, it suffices to prove that $P(q_i + q_j) \leq I(q_i, \bar{\pi})$ for $q_i \in (R^k - \epsilon, R^k + \epsilon)$. Hence, we only need to show $I(q_i, \bar{\pi}) - P(q_i + q_j)$ is locally a strictly convex function:

$$\frac{\partial^2}{\partial q_i^2}[I(q_i, \bar{\pi}) - P(q_i + q_j)] > 0 \Rightarrow \frac{\partial^2 P(q_i + q_j)}{\partial q_i^2} < \frac{\partial^2 I(q_i, \bar{\pi})}{\partial q_i^2}. \tag{14}$$

Because $P(q_i + q_j)$ is locally a strongly log-concave function, at the tangent point R^k which satisfies Eq. (13), we must have

$$\frac{\partial P(q_i + q_j)}{\partial q_i} + q_i \frac{\partial^2 P(q_i + q_j)}{\partial q_i^2} < 0 \Rightarrow \frac{\partial^2 P(q_i + q_j)}{\partial q_i^2} < -\frac{\partial P(q_i + q_j)}{\partial q_i} \frac{1}{q_i}. \tag{15}$$

Because $P(q_i + q_j)$ is twice continuously differentiable, Eq. (15) must also hold in the neighborhood of R^k. By Definition 1 and the expression of $I(q_i, \bar{\pi})$, we have

$$\frac{\partial P(q_i + q_j)}{\partial q_i} = \frac{\partial I(q_i, \bar{\pi})}{\partial q_i} = -\frac{\bar{\pi}}{q_i^2} = -\frac{\partial^2 I(q_i, \bar{\pi})}{\partial q_i^2} \frac{q_i}{2}. \tag{16}$$

Substituting Eq. (16) into the RHS of Eq. (15), and because $\partial^2 I(q_i, \bar{\pi})/\partial q_i^2$ is positive, $\partial^2 P(q_i + q_j)/\partial q_i^2 < \partial^2 I(q_i, \bar{\pi})/\partial q_i^2$.

QED

The standard assumptions guarantee that a global maximizer must be a local maximizer as well. Hence, each local maximizer $R^k(q_j)$ is potentially firm i's reaction function given q_j. For analytical simplicity, we only consider the potential existence of at most two types of local maximizers, labeled as $R^X(q_j)$ and $R^Y(q_j)$. The following lemma gives the conditions under which firm i's reaction function is discontinuous with one upward shifting point:[17]

Lemma 4 *In addition to the Standard Assumptions, assume that*

(i) $\exists q_j = \dot{q}_J$ *such that* $P(q_i, q_j)$ *and* $I(q_i, \bar{\pi})$ *are tangent at two points,* $R^X(q_j)$ *and* $R^Y(q_j)$.

(ii) $\exists \epsilon^k$ *such that* $P(q_i + q_j)$ *is strongly log-concave* $\forall q_i \in (R^k - \epsilon^k, R^k + \epsilon^k)$, *where* $k = X$ *or* Y.

Then firm i's reaction function $R_i(q_j)$ *is locally decreasing in* $q_j \leq \dot{q}_J$ *or* $q_j \geq \dot{q}_J$ *and jumps upward at* $\dot{q}_J$. *Formally*

$$R_i(q_j) = \begin{cases} R^X(q_j) & \text{if } q_j \leq \dot{q}_J, \\ R^Y(q_j) & \text{if } q_j \geq \dot{q}_J. \end{cases} \tag{17}$$

[17] If there potentially exist more than two types of local maximizers, we can derive the reaction function with more than two upward jumping points by a similar procedure.

Proof. Without losing generality, we assume $R^X(q_j) < R^Y(q_j)$ for all q_j.[18] The second assumption in Lemma 4 guarantees that both $R^X(q_j)$ and $R^Y(q_j)$ are decreasing in q_j. Next, we verify that $\dot{q}_J$ is the only solution to $\pi_i[R^X(q_j), q_j] = \pi_i[R^Y(q_j), q_j]$. This suffices to confirm the sign of

$$\frac{\partial}{\partial q_j}\{P[R^X(q_j) + q_j]R^X(q_j) - P[R^Y(q_j) + q_j]R^Y(q_j)\}. \tag{18}$$

The first order derivative of each term in the brackets with respect to q_j can be rearranged to

$$[\frac{\partial P(\cdot)}{\partial q_i}R^k(q_j) + P(\cdot)]\frac{\partial R^k(q_j)}{\partial q_j} + \frac{\partial P(\cdot)}{\partial q_i}R^k(q_j), \tag{19}$$

where $k = X$ or Y and the last term is derived from $\partial P(q_i + q_j)/\partial q_j = \partial P(q_i + q_j)/\partial q_i$. Substituting the tangent condition in Eq. (13), the first term in Eq. (19) is zero and the second term equals $-P[R^k(q_j) + q_j]$. Hence, Eq. (18) can be simplified as

$$-P[R^X(q_j) + q_j] + P[R^Y(q_j) + q_j] < 0. \tag{20}$$

The LHS of Eq. (20) is negative because $P(\cdot)$ is downward sloping. Therefore, $\dot{q}_J$ must be the only solution to $\pi_i[R^X(q_j), q_j] = \pi_i[R^Y(q_j), q_j]$, and $R^X(q_j)$ is selected as the global maximizer for $q_j \leq \dot{q}_J$.

QED

Since the game is symmetric, based on the above two types of correspondences, it is clear that when firms move simultaneously, we potentially have two types of Cournot equilibrium candidates, which are denoted by $(\hat{q}_c^X, \hat{q}_c^X)$ and $(\hat{q}_c^Y, \hat{q}_c^Y)$, with equilibrium profits π_c^X and π_c^Y.[19] Hence, theorems in the existing literature that assume a unique Cournot equilibrium (e.g., Hamilton and Slutsky, 1990) cannot apply in the current situation. In the sequential subgames, let the leader's interior equilibrium candidates be $\hat{q}_l^X$ and $\hat{q}_l^Y$, and the follower's be $R^X(\hat{q}_l^X)$ and $R^Y(\hat{q}_l^Y)$, respectively. The equilibrium profits are π_l^X, π_f^X, π_l^Y, π_f^Y. The leader and the follower may also have the corner equilibrium candidate, $\dot{q}_J$ and $R^X(\dot{q}_J)$ (as in the basic model), with equilibrium profits $\dot{\pi}_l^C$, $\dot{\pi}_f^C$. The following proposition summarizes the case wherein the strategic choices under the sequential moves becomes Pareto dominant ones over those under the simultaneous moves:

Proposition 2 *Suppose the standard assumptions and those in Lemma 4 hold, if* $\dot{q}_J < \hat{q}_c^X$ *and* $\dot{\pi}_l^C > \pi_l^Y$*, the strategic choices under the sequential moves Pareto dominate those under the simultaneous moves.*

[18] Because products are homogeneous, the change in q_j shifts $P(q_i + q_j)$ along the horizontal axis linearly, implying that the relative position of $R^X(q_j)$ and $R^Y(q_j)$ does not change with q_j.

[19] $\hat{q}_c^k$ is derived by solving $R^k(q_j) = q_j$, $k = X, Y$.

Proof. $\dot{q}_J < \hat{q}_c^X$ implies that under the simultaneous moves, the reaction function jumps upward before $\hat{q}_c^X$ can be reached. Hence $(\hat{q}_c^Y, \hat{q}_c^Y)$ is selected under the simultaneous moves. When the reaction function is given by $R^X(q_j)$, each firm's output level in the sequential game as a leader is strictly larger than that in the simultaneous game:

$$\hat{q}_l^X > \hat{q}_c^X. \tag{21}$$

Therefore, under the sequential moves, $\hat{q}_l^X$ cannot be reached before the follower's reaction function jumps upward. Together with the second inequality in Proposition 2, $(\dot{q}_J, R^X(\dot{q}_J))$ is selected under the sequential moves.

Because strategic variables are locally strategic substitutes, when the leader's quantity $\dot{q}_J$ is smaller than the Cournot level (under status X) $\hat{q}_c^X$, the follower's quantity $R^X(\dot{q}_J)$ must be larger than the leader's, implying

$$\dot{\pi}_f^C > \dot{\pi}_l^C. \tag{22}$$

When the reaction function is given by $R^Y(q_j)$, each firm's output level in the sequential game as a leader is strictly larger than that in the simultaneous game:

$$\hat{q}_l^Y > \hat{q}_c^Y \Rightarrow \pi_l^Y > \pi_c^Y. \tag{23}$$

Combining Eq. (22), Eq. (23) and the second inequality in Proposition 2, we obtain $\dot{\pi}_f^C > \dot{\pi}_l^C > \pi_l^Y > \pi_c^Y$.

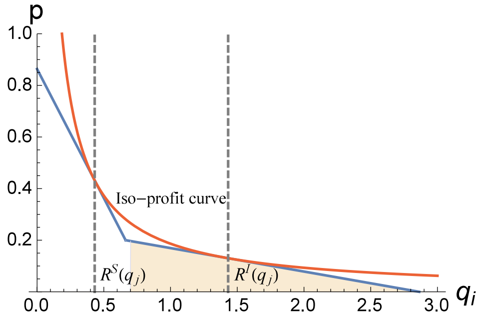
QED

The first inequality in Proposition 2 implies that choosing large outputs (based on $R^Y(\cdot)$) is attractive enough under the simultaneous moves. On the other hand, the second inequality implies that restricting total outputs (based on the jump point $\dot{q}_J$ and $R^X(\cdot)$) is better under the sequential moves. The condition in Proposition 2 is consistent with Proposition 1 (*ii*) in the sense that supplying consumers with low WTP is moderately profitable.

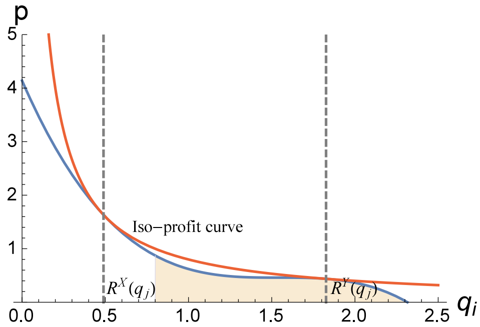
The cubic type demand function $P(q_1+q_2) = m+[n-(q_1+q_2)]^3$ is a good example of nonlinear demand structures. FIGURE 4 (*a*) depicts the kinked demand function with $a = 0.2$, $b = 2$ at $q_j = 0.137$. FIGURE 4 (*b*) depicts the cubic demand function with $m = 0.46$, $n = 2$ at $q_j = 0.456$. In each figure, there are two tangent points, implying that firm i changes its best response at the corresponding q_j. Both of them have the "long-tail" parts (shady areas), which come from strong consumer heterogeneity and a moderately large number of consumers whose WTP is very low.

5.1 *Example*

We now discuss the endogenous timing problem under the cubic demand function, $P(q_i + q_j) = 0.46 + [2 - (q_i + q_j)]^3$. Each firm's reaction function is derived by solving



(a) The kinked demand function



(b) The cubic demand function

FIGURE 4 The cubic demand function

$P(q_i + q_j) = I(q_i, \bar{\pi})$ and $\partial P(q_i + q_j)/\partial q_i = \partial I(q_i, \bar{\pi})/\partial q_i$:[20]

$$R_i(q_j) = \begin{cases} R^X(q_j) = \dfrac{(2-q_j)[3-(cos\frac{\theta}{3}+\sqrt{3}sin\frac{\theta}{3})]}{4} & if\ q_j \leq \dot{q}_J \equiv 0.456, \\ R^Y(q_j) = \dfrac{(2-q_j)+(\sqrt[3]{\xi_\alpha}+\sqrt[3]{\xi_\beta})}{12} & if\ q_j \geq \dot{q}_J, \end{cases} \tag{24}$$

where $\theta = arccos\dfrac{q_j^3 - 6q_j^2 + 12q_j - 4.32}{(2-q_j)^3}$; $\xi_{\alpha,\beta} = 81(2-q_j)^3 + 6(B \pm \sqrt{B^2 - 4AC})$;

$A = 9(2-q_j)^2$; $B = 18q_j^3 - 108q_j^2 + 216q_j - 127.44$; $C = 9q_j^4 - 72q_j^3 + 216q_j^2 - 275.58q_j + 119.16$. $q_j = \dot{q}_J$ solves $\pi_i[q_j, R^X(q_j)] = \pi_i[q_j, R^Y(q_j)]$. At this point, i's reaction function has an upward jump from 0.489 to 1.826, from which the corresponding profit is $\pi_i = 0.799$. We can confirm that $-1 < (q_j^3 - 6q_j^2 + 12q_j - 4.32)/(2-q_j)^3 < 1$, $B^2 - 4AC > 0$, and that both $R^X(q_j)$ and $R^Y(q_j)$ are continuously sloping down, in the corresponding ranges of q_j.

In the sequential moving case, as the follower's reaction function upward shifts at $q_j = \dot{q}_J$, at this point, the market price has a sudden drop from 1.633 to 0.437. Given the follower's reaction function, the leader's profit curve is depicted in FIGURE 5. In this case, the leader's profit at $q_j = \dot{q}_J$ is higher than the locally optimal one at around 2.0, suggesting that $\dot{q}_J$ is the leader's globally optimal output. Then, $(q_l, q_f) = (\dot{q}_J, R^X(\dot{q}_J))$ is the equilibrium outputs in the sequential moving case. The corresponding profits are $\pi_l = 0.745$, $\pi_f = 0.799$, implying the existence of the second mover advantage.

In the simultaneous moving case, by symmetry, we derive the equilibrium outputs by solving $R_i(q_j) = q_j$. The solutions must satisfy the second assumption in Lemma 4. Hence, we derive two equilibrium candidates: $q_c^X = 0.488$ and $q_c^Y = 1.173$. The first candidate must be eliminated because before it is reached the reaction function has already upward jumped. Therefore, $(q_i, q_j) = (1.173, 1.173)$ is the only equilibrium

[20] The reaction function is derived by using Shengjin's formulas (Fan 1989).

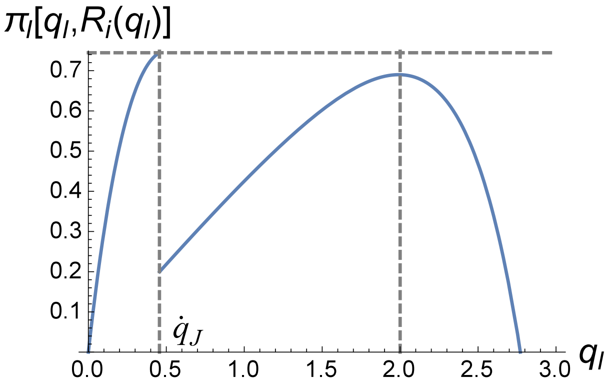


FIGURE 5 The leader's profit curve

outcome in the simultaneous moving game. The corresponding profit is $\pi_c = 0.491$.

Because $\pi_f > \pi_l > \pi_c$, the sequential timing outcomes exist. The firms' strategy choices under the sequential moves Pareto dominate those under the simultaneous moves.

6 Concluding remarks and discussion

This study indicates the manner in which firms' timing is decided when facing consumers who are heterogeneous in WTP. If the size of the low-end market is moderately small, firms may choose to move sequentially with an outcome that the follower earns a higher profit than the leader. Owing to the existence of heterogeneous consumers, each firm is given an option to choose a large output and reduce the price such that it can enter the low-end market. Thus, each firm's reaction function has an upward jump if its rival oversupplies. This behavior significantly affects the equilibrium outcome when firms move sequentially. If the leader does not restrict its output to lower than that of the follower, the follower raises its output substantially so that price collapse occurs. Thus, the firm that acts as a follower earns a higher profit than the leader.

We generalize our basic model to a general demand case. We demonstrate that the inverse demand functions that are sufficiently non-concave may give rise to multiple equilibrium candidates in the quantity-setting duopoly competition, leading to upward shifting reaction functions. The sequential timing outcomes stem from discontinuous reaction functions and may exist in other nonlinear demand structures such as the cubic demand case. We believe the driving force of our results comes from consumers' large income inequality. When entering developing countries where consumer income inequality is relatively pronounced, firms have to adopt inconsistent strategies while responding to their rivals' choices.

For simplicity, we only consider the case of symmetric and constant marginal costs (both are zero) and homogeneous products. Notice that the essential driving force behind our results is each firm's discontinuous reaction function which has an upward shift point. In the general demand case, if we assume one firm has a cost disadvantage compared with its rival (the former one with constant and positive marginal cost while the later with zero marginal cost), then the marginal cost would vertically move the iso-profit curves $I(q_i, \bar{\pi})$ upward, leading to a leftward shift of the cost disadvantaged firm's local maximizers R^k. Moreover, if we assume products are heterogeneous with an inverse demand function $P_i(q_i + \gamma q_j)$ where $0 < \gamma < 1$ is the substitutability between different varieties, Eq. (18) can be simplified as

$$-\gamma P[R^X(q_j) + q_j] + \gamma P[R^Y(q_j) + q_j] < 0. \tag{25}$$

Hence, the condition for an upward shifting reaction function can be derived in a way analogous to that in Section 5. With such reaction functions, we can still obtain sequential moving outcomes if the inequalities in Proposition 2 (with modification) hold.

Appendix

The inverse demand

Let $D^H(P)$ and $D^L(P)$ be the demand functions of the high- and low-end markets which are given as follows:

$$D^H(P) = \max\{1 - P, 0\}, \tag{A1}$$

$$D^L(P) = \max\{b(1 - \frac{P}{a}), 0\}. \tag{A2}$$

If $1 - q_1 - q_2 \geq a$,

$$P = 1 - D^H(P) = 1 - (q_1 + q_2). \tag{A3}$$

If $0 < 1 - q_1 - q_2 < a$,

$$D^H(P) + D^L(P) = 1 - P + b(1 - \frac{P}{a}) \Rightarrow P = \frac{a(1 + b - q_1 - q_2)}{a + b}. \tag{A4}$$

Firms' reaction functions

By solving the profit maximization problem in Eq. (2), we obtain the reaction function as follows:

$$R_i(q_j) = \begin{cases} R^S(q_j) & if\ q_i + q_j \leq 1 - a, \\ R^I(q_j) & if\ q_i + q_j > 1 - a. \end{cases} \tag{A5}$$

Since $R^S(q_l)$ does not always satisfy the first conditional formula in Eq. (A5), the existence type S outcome needs to satisfy $R^S(q_j) + q_j \leq 1 - a$. By arranging the inequality, we obtain

$$q_j \leq 1 - 2a. \tag{A6}$$

For the existence of type I outcome, we need $R^I(q_j)$ to satisfy the second conditional formula in Eq. (A5): $R^I(q_j) + q_j > 1 - a$. By arranging the inequality, we obtain

$$q_j > 1 - b - 2a. \tag{A7}$$

For $1 - b - 2a < q_j \leq 1 - 2a$, the firm i chooses $R^S(q_j)$ if this brings it a higher profit, or $\pi_i^S[q_j, R^S(q_j)] \geq \pi_i^I[q_j, R^I(q_j)]$. By arranging the inequality, we obtain

$$q_l \leq q_J. \tag{A8}$$

It is straightforward that $1 - b - 2a < q_J \leq 1 - 2a$ for any $0 < a < 1$, $b > 0$. Therefore, firm i chooses $R^S(q_j)$ for $q_j \leq q_J$. Thus, we obtain the reaction function in Eq. (3).

Proof for Lemma 1

Proof. From the reaction function in Eq. (3), due to symmetry, two types of equilibrium candidates, as in Eq. (4), can be derived by solving $R^S(q_j) = q_j$ and $R^I(q_j) = q_j$. $(\hat{q}_c^S, \hat{q}_c^S)$ exist when

$$\frac{1}{3} \leq q_J \Rightarrow b \leq \frac{4 - 12a}{9a}. \tag{A9}$$

$(\hat{q}_c^I, \hat{q}_c^I)$ exist when

$$\frac{1 + b}{3} \geq q_J \Rightarrow b \geq \frac{4 + 3a - 3\sqrt{8a + a^2}}{2}. \tag{A10}$$

For $(4 + 3a - 3\sqrt{8a + a^2})/2 \leq b \leq (4 - 12a)/(9a)$, firms coordinate to the type S equilibrium outcome if

$$\pi_c^S \geq \pi_c^I \Rightarrow b \leq \frac{1 - 2a}{a}. \tag{A11}$$

Because $(1 - 2a)/a \geq (4 - 12a)/(9a)$, type S is always picked up for $(4 + 3a - 3\sqrt{8a + a^2})/2 \leq b \leq (4 - 12a)/(9a)$. In other words, as long as $(\hat{q}_c^S, \hat{q}_c^S)$ exists, it is picked up as the equilibrium outcome. Thus, we obtain Lemma 1.

QED

Proof for Lemma 2

Proof. When q_l^S is selected by the leader, q_l^S must satisfy the first conditional inequality in Eq. (3):

$$\frac{1}{2} \leq q_J \Rightarrow b \leq \frac{1 - 4a}{4a}. \tag{A12}$$

If the inequality in Eq. (A12) is satisfied, choosing q_J must be a dominated strategy because q_l^S is the local optimal quantity. Besides, we also require that q_l^I does not satisfy the second conditional inequality in Eq. (3) or choosing q_l^I is not as profitable as choosing q_l^S for the leader:

$$\frac{1+b}{2} < q_J \; or \; \pi_l^S \geq \pi_l^I \Rightarrow b \leq \frac{1-2a}{a}. \tag{A13}$$

Combining Eq. (A12) and Eq. (A13), we derive the inequality in Lemma 2 (*i*).

When the corner equilibrium candidate q_J is selected by the leader, q_l^S must not satisfy the first conditional inequality in Eq. (3), otherwise q_l^S would be selected. Besides, we require that choosing q_l^I is not as profitable as choosing q_J for the leader:

$$\frac{1}{2} > q_J \; and \; \pi_l^C \geq \pi_l^I \Rightarrow \frac{1-4a}{4a} < b \leq b(a). \tag{A14}$$

Then, we derive the inequality in Lemma 2 (*ii*).

When q_l^I is selected by the leader, q_l^I must satisfy the first conditional inequality in Eq. (3):

$$\frac{1+b}{2} \geq q_J \Rightarrow b \geq 1 - 2\sqrt{a}. \tag{A15}$$

Besides, we need exclude the case that q_l^S or q_J is selected. If q_l^S is selectable, we require that choosing q_l^S is not as profitable as choosing q_l^I. On the other hand, if q_l^S is not selectable, we require that choosing q_J is not as profitable as choosing q_l^I :

$$\left.\begin{array}{l} \frac{1}{2} \leq q_J \; and \; \pi_l^S \leq \pi_l^I, \\ or \; \frac{1}{2} \geq q_J \; and \; \pi_l^C \leq \pi_l^I \end{array}\right\} \Rightarrow b > b(a). \tag{A16}$$

Combining Eq. (A15) and Eq. (A16), we derive the inequality in Lemma 2 (*iii*).

QED

Proof for Proposition 1

Proof. (*i*) If $0 < b \leq (4-12a)/(9a)$, the market always separates, whether firms move simultaneously or sequentially. The lower bound for the existence of $(q_J, (1-q_J)/2)$ equilibrium outcome, $(1-4a)/(4a)$, locate between 0 and $(4-12a)/(9a)$. We first consider $0 < b \leq (1-4a)/(4a)$. In the simultaneous subgames, both firms obtain π_c^S. In the sequential subgames, the leader and the follower obtain π_l^S and π_f^S respectively. Because $\pi_l^S > \pi_c^S > \pi_f^S$, firms move simultaneously in equilibrium. Next, we consider $(1-4a)/(4a) < b \leq (4-12a)/(9a)$. In the simultaneous subgames, both firms obtain π_c^S. In the sequential subgames, the leader and the follower obtain π_l^C and π_f^C respectively. Because $\pi_l^C > \pi_c^S > \pi_f^C$, firms move simultaneously in equilibrium. Therefore, we obtain Proposition 1 (*i*).

(*ii*) If $(4-12a)/(9a) < b \leq b(a)$, in the simultaneous subgames, both firms obtain π_c^I and the market integrates. In the sequential subgames, the leader and the follower obtain π_l^C and π_f^C respectively, and the market separates. Because $\pi_f^C > \pi_l^C > \pi_c^I$, firms move sequentially in equilibrium. Therefore, we obtain Proposition 1 (*ii*).

(*iii*) If $b > b(a)$, the market always integrates, whether firms move simultaneously or sequentially. In the simultaneous subgames, both firms obtain π_c^I. In the sequential subgames, the leader and the follower obtain π_l^I and π_f^I respectively. Because $\pi_l^I > \pi_c^I > \pi_l^I$, firms move simultaneously in equilibrium. Therefore, we obtain Proposition 1 (*iii*).

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